SSIE 501

Practice Problems (mathematical ones)

1. Given *X* = {1, 2, 3, 4, 5, 6, 7}, *R*  *X*2

*R* = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2),

(3, 3), (3, 4), (4, 1), (4, 3), (4, 4), (5, 5), (5, 6), (5, 7), (6, 5),

(6, 6), (6, 7), (7, 5), (7, 6), (7, 7)}

1. Express *R* as a matrix.
2. Express *R* graphically (as a directed graph).
3. Is *R* reflexive? Why or why not?
4. Is *R* anti-reflexive? Why or why not?
5. Is *R* symmetric? Why or why not?
6. Is *R* anti-symmetric? Why or why not?
7. Is *R* transitive? Why or why not?
8. Is *R* an equivalence relation? Why or why not?
9. Is *R* a compatibility relation? Why or why not?
10. Is *R* a partial ordering? Why or why not?

2. Given *X* = {1, 2, 3, 4},

*R* P (*X*) × P (*X*) where P (*X*) denotes the power set of *X.*

Specifically for any sets *A* and *B* which are elements of P (*X*),

(*A, B*)  *R* if and only if *B*  *A.*

1. Specifically, what is P (*X*)?
2. Write out *R* as a set of ordered pairs.
3. Express *R* as a matrix.
4. Express *R* graphically (as a directed graph).
5. Is *R* reflexive? Why or why not?
6. Is *R* anti-reflexive? Why or why not?
7. Is *R* symmetric? Why or why not?
8. Is *R* anti-symmetric? Why or why not?
9. Is *R* transitive? Why or why not?
10. Is *R* an equivalence relation? Why or why not?
11. Is *R* a compatibility relation? Why or why not?
12. Is *R* a partial ordering? Why or why not?

3. Given *X* = {1, 2, 3, 4}, *Y* = {*a, b, c*}, *Z* = {}

1. What is *#R*(*X* × *Y*)?
2. Make up your own example of a relation based on *X* × *Y*. Express the relation as a set of ordered pairs, as a matrix, and graphically (using two boxes and lines between them—a mapping diagram).
3. What is *#R*(*X* × *Y*2 × *Z*)?
4. Make up your own example of a relation based on *X* × *Y*2 × *Z*. (Express it only as a set of ordered 4-tuples. You need not express it as a matrix or graphically.)
5. What is *#R*(P (*X*)× *Y* × *Z*)?
6. Make up your own example of a relation based on P (*X*)× *Y* × *Z*. (Express it only as a set of ordered 3-tuples. You need not express it as a matrix or graphically.)

4. You are given the following three systems:

*S*1 = (*T*1, *R*1), *T*1 = {1, 2, 3, 4, 5, 6, 7}, *R*1  *T*12,

*R*1 = {(1, 2), (1, 3), (2, 3), (2, 4), (3, 2), (4, 5), (4, 6), (5, 3), (5, 4),

(6, 4), (6, 7), (7, 5)}

*S*2 = (*T*2, *R*2), *T*2 = {*a, b, c, d, e, f, g*}, *R*2  *T*22,

*R*2 = {(*a, b*), (*a, c*), (*b, a*), (*b, e*), (*c, a*), (*c, d*), (*d, b*), (*e, f*), (*f, a*),

(*f, e*), (*g, e*), (*g, f*)}

*S*3 = (*T*3, *R*3), *T*3 = {}, *R*3  *T*32,

*R*3 = {(), (), (), (), (), (), (), (), (),

(), (), ()}

1. Are *S*1 and *S*2 isomorphic systems? Explain clearly why or why not.
2. Are *S*1 and *S*3 isomorphic systems? Explain clearly why or why not.
3. Are *S*2 and *S*3 isomorphic systems? Explain clearly why or why not.

5. Given S is the set of all possible systems based on Klir’s common-sense definition of systems. (This is obviously an infinite set.)

Let *Ri*  S 2 specifically where for any systems *S*1 and *S*2  S ,

(*S*1 , *S*2)  *Ri* if and only if *S*1 and *S*2 are isomorphic systems. Explain in your own words why *Ri* is an equivalence relation. (If you cannot write this as a formal proof at least explain in detail the basis of a proof.)

6. Consider two systems, *S*1 =(*T*1, *R*1) and *S*2 = (*T*2, *R*2). Let *T*1 represent a set of nurses who have worked in a particular ward in a particular hospital. Here, we will just anonymously label them Nurse #1 through Nurse #10. The nurses’ shift assignments vary from week to week, so that some nurses work on the same schedule one week and may work with other nurses the next. Let *R*1 ⊆ *T*12 be a relation defined such that for any *x, y* ∈ *T*1, let (*x, y*) ∈ *R*1 if and only if Nurse *x* has at some point worked on the same shift assignment as Nurse *y*. We are also provided with the following information.

Nurse #1 has at some time worked a shift with Nurses# 2, 3, 4, and 5 (not necessarily all at the same time).

Nurse #2 has at some time worked a shift with Nurses# 4 and 5.

Nurse #3 has at some time worked a shift with Nurses# 4 and 6.

Nurse #4 has at some time worked a shift with Nurses# 5, 6, 7, and 8.

Nurse #5 has at some time worked a shift with Nurses# 7 and 8.

Nurse #6 has at some time worked a shift with Nurses# 7 and 9.

Nurse #7 has at some time worked a shift with Nurses# 8, 9, and 10.

Nurse #8 has at some time worked a shift with Nurse#10.

Nurse #9 has also at some time worked a shift with Nurse#10.

Let *T*2 represent a set of 10 prescription drugs, and we’ll just label these Drug *a* through Drug *j*. Let *R*2 ⊆ *T*22 be a relation defined such that for any *x, y* ∈ *T*2, let (*x, y*) ∈ *R*2 if and only if Drug *x* is taken together with Drug *y* by at least one patient. We also know the following.

Drug *a* is sometimes taken together with Drugs *d, f, i,* and *j* (not necessarily all by the same patient).

Drug *b* is sometimes taken together with Drugs *c, d, g, h,* and *j.*

Drug *c* is sometimes taken together with Drugs *d, g,* and *i.*

Drug *d* is sometimes taken together with Drugs *g, h, i* and *j.*

Drug *e* is sometimes taken together with Drugs *f, h,* and *j*.

Drug *f* is sometimes taken together with Drug *j.*

Drug *h* is also sometimes taken together with Drug *j.*

1. Is *R*1 reflexive, anti-reflexive, symmetric, anti-symmetric, transitive, an equivalence relation, a compatibility relation, a partial ordering? In each case, be prepared to justify your answer. (Hint: Keep in mind that there may be implicit aspects to how the relations are defined.)
2. Is *R*2 reflexive, anti-reflexive, symmetric, anti-symmetric, transitive, an equivalence relation, a compatibility relation, a partial ordering? In each case, be prepared to justify your answer.
3. Are *S*1 and *S*2 isomorphic systems? Justify your answer in detail.